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Wave-power absorption by systems of oscillating surface pressure distributions

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Some general results are derived for the efficiency of energy absorption of a system of uniform oscillatory surface pressure distributions. The results, which are based on classical linear water-wave theory, show the close analogies which exist with theories for systems of absorbing oscillatory rigid bodies and a number of new reciprocal relations for pressure distributions are suggested and proved. Some simple examples illustrating the general results are given and compared with the corresponding results for rigid bodies.

1. Introduction

A number of wave-energy devices currently being considered both in the U.K. and elsewhere base their mode of operation on the following principle. A region of the free surface is surrounded by a rigid hollow floating structure, open at the immersed bottom end, which traps a volume of air above this internal free surface. The incident wave field causes a rise and fall of the free surface and the volume of air is driven back and forth at high speed through a constriction containing an air turbine feeding a generator for direct conversion to electricity. At model scale the air turbine is replaced by a simple orifice plate – the size of the orifice being adjusted to attempt to match the full-scale turbine characteristics. Examples of devices which operate on this principle are the C.E.G.B. device (Count *et al.* 1981), the buoy being developed by Queens University, Belfast, and the Kaimei device being tested in Japan. Descriptions of these last two devices can be found in Quarrell (1978).

In an attempt to model the hydrodynamics of such devices, authors have adapted theory developed for wave-energy devices involving rigid oscillating bodies and described, for example, in Evans (1981). This usually involves replacing the free surface by a weightless piston and requires the determination of the added mass and damping of the piston. Examples of this approach, which neglects any spatial variation in the internal free surface caused by the surface pressure, include Evans (1978), who considers the resonant oscillations of a narrow water column in an immersed open vertical tube, and Count *et al.* (1981) who compute the hydrodynamical coefficients for the C.E.G.B. wave energy device, which can be aptly described as a half-open matchbox floating upside-down on the water surface.

This paper presents a more accurate yet simpler theory for such devices which correctly allows for the applied surface pressure and the consequent spatial variation of the internal free surface.

A similar approach to the two-dimensional wave-energy problem has been made by Falcão & Sarmento (1980), extending work by Stoker (1957). The present work generalizes their results to arbitrary pressure distributions in both two and three dimensions. In a different context Ogilvie (1969) has also considered some two-dimensional problems involving pressure fields. He obtains results which he uses to predict the motion of a long air-cushion vehicle with side walls. He also solves explicitly the difficult problem of a uniform pressure field over a segment of the surface bounded by two equally submerged vertical plates. No computations of the solution are made.

In §2 the general theory is developed and results for the hypothetical maximum power absorption of an arbitrary system of energy-absorbing pressure distributions are derived. The close resemblance to corresponding results from rigid-body theory suggests various reciprocal relations between certain radiation and scattering problems which are duly proved in the appendix. Simple special cases which illustrate the general theory are presented in both §2 and §3 where in addition more practical considerations provide conditions for resonance. In §4 a comparison with the rigid-body resonance conditions is made and curves of the maximum absorption width in each case for a single circular surface pressure or rigid disk are presented.

2. Formulation

To fix ideas we consider a fixed structure open at the bottom end, and closed at the top end, which intersects the free surface, trapping a volume of air in a series of separate sections each having its own internal free surface. The effect of an incident wave train is to cause the internal free surfaces to oscillate at the same frequency as the incident wave, driving their air volumes back and forth through constrictions containing turbines. It is assumed that the compressibility of the air is small so that the air pressure at each turbine is the same as the uniformly distributed pressure just above the corresponding free surface. The total mean rate of doing work will be the sum of time averages of the product of these pressures and the volume flows through the turbines, which in turn is the same as the product of the spatial average of the vertical velocity of each internal free surface and its area. In the present work we assume that the turbine characteristics are linear so that the pressure drop across the turbine is proportional to the volume flow through it. We take Cartesian co-ordinates with x, y horizontal, and z vertically upwards, with $z = 0$ the undisturbed free surface.

Under the assumptions of linear water-wave theory, we can construct a velocity potential $\Phi(x, y, z, t)$ for the problem satisfying

$$\nabla^2\Phi = 0 \quad \text{in the fluid,} \tag{2.1}$$

$$g\eta + \frac{\partial\Phi}{\partial t} = \begin{cases} \dfrac{-P_i(t)}{\rho} & \text{on } S_i, \\ 0 & \text{on } S_F, \end{cases} \tag{2.2}$$

where S_i is the ith internal free surface, S_F is the external free surface, and $\eta(x, y, t)$ is the surface elevation satisfying

$$\frac{\partial\eta}{\partial t} = \frac{\partial}{\partial z}\Phi(x, y, 0, t), \tag{2.3}$$

$$\frac{\partial\Phi}{\partial n} = 0 \quad \text{on rigid boundaries, } S_B. \tag{2.4}$$

Here $P_i(t)$ is the, as yet unknown, simple harmonic pressure on S_i. The effect of the structure is partially to scatter the incident waves so that, at large distances, in addition to the incident wave potential, there exists a wave field travelling outwards away from the structure.

The incident wave potential can be described by

$$\Phi_0(x, y, z, t) = gA\omega^{-1} e^{kz} \cos(kx\cos\beta + ky\sin\beta - \omega t), \tag{2.5}$$

where $k = \omega^2/g$, and the incident wave-train makes an angle β with the positive x axis.

Equations (2.2) and (2.3) can be combined to give

$$\frac{\partial \Phi}{\partial z} - k\Phi = -\frac{1}{\rho g}\frac{dP_i}{gt} \tag{2.6}$$

for simple harmonic motions.

It is convenient to write

$$\Phi = \Phi_d + \Psi, \tag{2.7}$$

where Φ_d denotes a scattered plus incident potential satisfying (2.1), (2.4), and (2.6) with $P_i(t) = 0$ and Ψ is a radiation potential satisfying (2.1), (2.4) and (2.6) but which behaves like outgoing waves at large distances. It is clear then that Φ as given by (2.7) will satisfy all the conditions of the problem.

Now the volume flow rate across S_i is just

$$\begin{aligned}\int_{S_i} \frac{\partial \Phi}{\partial z}\,dS &= \int_{S_i} \frac{\partial \Phi_d}{\partial z}\,dS + \int_{S_i} \frac{\partial \Psi}{\partial z}\,dS \\ &= Q_{di}(t) + Q_i(t), \quad \text{say.}\end{aligned}$$

The total rate of working of the pressure forces across all S_i is then

$$\sum_{i=1}^{N} P_i(t)\,(Q_{di}(t) + Q_i(t)) = \mathbf{P}^T(\mathbf{Q}_d + \mathbf{Q}), \tag{2.8}$$

where $\mathbf{P}$, $\mathbf{Q}_d$, $\mathbf{Q}$, are column vectors whose ith components are P_i, Q_{di} and Q_i respectively. Now the simple harmonic pressure $P_i(t)$ at S_i alone will give rise to volume flow rates $Q_j(t)$ on S_j ($j = 1, \ldots, N$), which are also simple harmonic in time. We make the arbitrary but convenient decomposition

$$\mathbf{Q} = -\mathbf{A}\dot{\mathbf{P}} - \mathbf{B}\mathbf{P} \tag{2.9}$$

where $\mathbf{A}$, $\mathbf{B}$ are $N \times N$ real symmetric matrices, with B a damping coefficient, positive definite in general, which can, in principle, be determined. The decomposition (2.9) can be compared to the usual decomposition of the force on an oscillating body in terms of added-mass and damping matrices.

It is convenient at this stage to introduce time-independent quantities. We write

$$\{\Phi, \Phi_d, \Psi, \mathbf{P}, \mathbf{Q}, \mathbf{Q}_d\} = \mathscr{R}\{\phi, \phi_d, \psi, \mathbf{p}, \mathbf{q}, \mathbf{q}_d\}\, e^{-i\omega t}.$$

Then averaging over a period, the *mean* rate of working of the pressure forces becomes

$$W = \tfrac{1}{2}\mathscr{R}\mathbf{p}^*(\mathbf{q}_d + \mathbf{q}). \tag{2.10}$$

Here $*$ denotes conjugate transpose.

In terms of time-independent quantities, (2.9) may be written

$$\mathbf{q} = -\mathbf{Z}\mathbf{p}, \quad \text{where} \quad \mathbf{Z} = \mathbf{B} - i\omega\mathbf{A} \tag{2.11}$$

is a complex admittance. Then (2.10) becomes

$$W = \tfrac{1}{2}\mathscr{R}\mathbf{p}^*\mathbf{q}_d - \tfrac{1}{2}\mathbf{p}^*\mathbf{B}\mathbf{p}, \tag{2.12}$$

where we note that $\mathbf{A}$ does not appear. The expression (2.12) can be re-written in the form

$$W = \tfrac{1}{8}\mathbf{q}_d^*\mathbf{B}^{-1}\mathbf{q}_d - \tfrac{1}{2}(\mathbf{p} - \tfrac{1}{2}\mathbf{B}^{-1}\mathbf{q}_d)^*\mathbf{B}(\mathbf{p} - \tfrac{1}{2}\mathbf{B}^{-1}\mathbf{q}_d). \tag{2.13}$$

It follows that, provided $\mathbf{B}^{-1}$ exists,

$$W_{\max} = \tfrac{1}{8}\mathbf{q}_d^*\mathbf{B}^{-1}\mathbf{q}_d, \tag{2.14}$$

when

$$\mathbf{p} = \tfrac{1}{2}\mathbf{B}^{-1}\mathbf{q}_d. \tag{2.15}$$

The maximum mean power, then, would be achieved by ensuring that the pressure at, say, S_i is a linear combination of the volume fluxes induced at each S_j, $j = 1, \ldots, N$, due to the incident plus scattered wave alone, the constants of proportionality being such that (2.15) is satisfied. The results (2.14) and (2.15) are identical with the corresponding expressions obtained for a system of independently oscillating absorbing bodies in a regular incident wave-train. The roles of pressure and incident wave-induced volume flux are then replaced by velocity and incident-wave exciting force on the bodies.

Now in practice it may be easier to control the volume flux through the turbines than the pressure drop across. We shall assume a *linear* relation between them of the form

$$\mathbf{q} + \mathbf{q}_d = +\mathbf{\Lambda}\mathbf{p}, \tag{2.16}$$

where $\mathbf{\Lambda}$ is an $N \times N$ matrix. Notice that the sign in front of $\mathbf{\Lambda}$ is taken to be positive since, in contrast to (2.11), the pressure forces and volume fluxes are both measured vertically upwards. This, when used in conjunction with (2.11), gives

$$(\mathbf{\Lambda} + \mathbf{Z})\mathbf{p} = +\mathbf{q}_d, \tag{2.17}$$

which from (2.15) shows that

$$\mathbf{\Lambda} = \bar{\mathbf{Z}} \tag{2.18}$$

for maximum power. Here a bar denotes complex conjugate. In fact (2.13) can be written, after some manipulation,

$$W = \tfrac{1}{8}\mathbf{q}_d^*\{\mathbf{B}^{-1} - \mathbf{E}^*\mathbf{B}^{-1}\mathbf{E}\}\mathbf{q}_d, \tag{2.19}$$

where

$$\mathbf{E} = (\mathbf{Z} - \mathbf{\Lambda})(\mathbf{\Lambda} + \mathbf{Z})^{-1}, \tag{2.20}$$

and (2.17) has been used. This form exhibits clearly the impedance matching condition required for optimality.

Unfortunately, in reality, unless the turbines are linked, each turbine will have its own pressure/flow characteristic, here assumed linear, so that $\mathbf{\Lambda}$ will be a diagonal matrix whereas both $\mathbf{B}$ and $\mathbf{A}$ are full matrices. Furthermore, unless the pump characteristics exhibit a phase lag between pressure and volume flow, the elements of $\mathbf{\Lambda}$ will be real and positive. Thus in a particular case the expression (2.19) needs to be maximized as a function of the positive turbine characteristics represented by the non-zero elements λ_i of $\mathbf{\Lambda}$. Even in the simplest case when all λ's are identical there does not appear any obvious analytical method of maximizing (2.19) and numerical optimization must be used.

The way to proceed in a particular problem is now clear. First the complex admittance matrix $\mathbf{Z}$ must be determined, either theoretically or experimentally. This involves finding the volume flux induced across all the S_j due to a uniform pressure distribution over each S_j in turn. Next $\mathbf{q}_d$, the induced volume flux across each S_j due to the diffracted potential alone, must be determined and finally the mean power absorbed can be obtained from (2.19) in terms of the (assumed linear) turbine characteristics modelled by the matrix $\mathbf{\Lambda}$.

Before looking at specific examples it is of interest to consider the *theoretical* maximum power under the assumption that the impedances can be matched exactly. In other words we assume there is a control mechanism which ensures that $\mathbf{\Lambda} = \bar{\mathbf{Z}}$ so that the volume flux across S_j is a predetermined linear combination of the pressures at each S_j. In this case

$$W_{\max} = \tfrac{1}{8}\mathbf{q}_d^* \mathbf{B}^{-1}\mathbf{q}_d, \tag{2.21}$$

a result having a parallel in the theory for systems of independently oscillating absorbing rigid bodies (Evans 1979; Falnes 1980), where $\mathbf{q}_d$ is replaced by the exciting force vector on the system of bodies.

Now it is shown in the appendix equation (A 27) that the elements of $\mathbf{B}$ are related to the elements of the vector $\mathbf{q}_d$ so that it is only necessary to determine the induced volume flux across each S_i due to the diffracted wave potential as in (A 17) in order to obtain the elements of the damping matrix $\mathbf{B}$ and consequently $W_{\max}$ from (2.21). Now the diffracted potential arises due to the presence of any fixed rigid structure and is independent of the pressure distributions. Thus ϕ_d and hence q_{di} may be determined using existing diffraction programs common in ship hydrodynamic theory.

A further simplification is possible if it can be assumed that the fixed immersed part of the absorber is of shallow draught. Then the only hydrodynamical effect of the structure is to limit the size and shape of the internal free surfaces S_i. But now the evaluation of q_d is trivial since it requires only the integration of the *incident* potential over S_i since the scattered potential can be neglected.

As an example of the general theory we consider just a single internal free surface S_1 so that

$$W_{\max}(\beta) = \tfrac{1}{8}|q_d|^2/B, \tag{2.22}$$

where from (A 1)

$$B = \frac{1}{8\lambda P_W}\int_0^{2\pi} |q_d(\theta)|^2\, d\theta. \tag{2.23}$$

If S_1 and S_B are axisymmetric so that q_d is independent of angle of incidence, we obtain

$$l_{\max}(\beta) \equiv W_{\max}(\beta)/P_W = \frac{\lambda}{2\pi} = k^{-1}, \tag{2.24}$$

where $l(\beta)$ is defined to be a capture width for the device and λ is the incident wavelength. This result is identical with that obtained for axisymmetric single absorbing bodies in heave (Budal & Falnes 1975; Evans 1976; Newman 1976).

For non-axisymmetric pressure distributions having zero draft, further progress can still be made using (2.23).

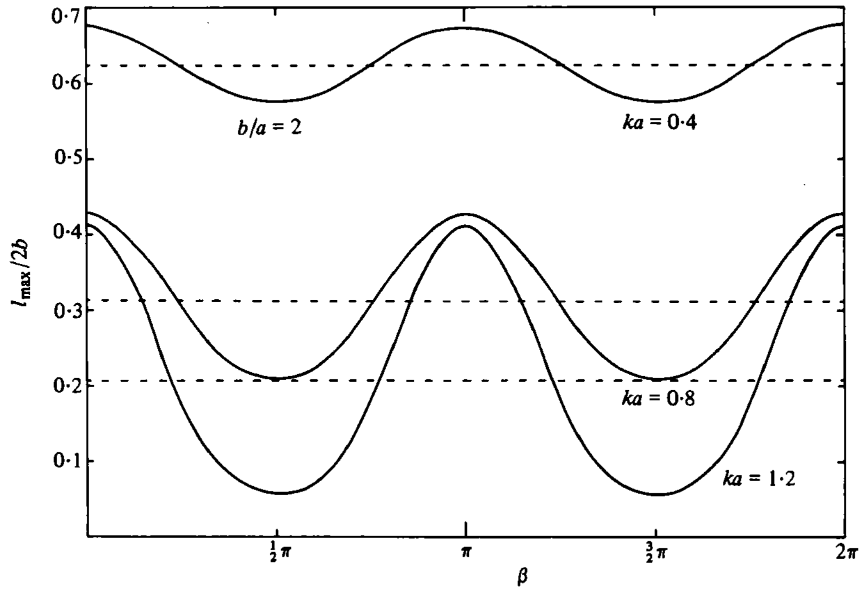

FIGURE 1. Variation of maximum capture width ratio, $l_{max}/2b$, with angle of incidence β, for regular waves approaching a rectangular absorbing surface pressure distribution, for different values of dimensionless wavenumber ka and $b/a = 2$. The dotted lines show the axisymmetric values $(2kb)^{-1}$.

We have from (2.22)–(2.24)

$$kl_{\max}(\beta) = |q_d(\beta)|^2 \bigg/ \int_0^{2\pi} |q_d(\theta)|^2 \, d\theta. \tag{2.25}$$

Consider a single rectangular pressure distribution with zero draught, occupying S_1: $|x| \leqslant a$, $|y| \leqslant b$. Then

$$q_d(\beta) = \int_{S_1} \{\partial\phi_0/\partial z\}_{z=0} \, dS,$$

where

$$\phi_0 = gA\omega^{-1} \exp\{ikx\cos\beta + iky\sin\beta + kz\},$$

so that

$$q_d(\beta) \equiv 4gA\omega^{-1}k^{-1}f(\beta),$$

where

$$f(\beta) = \sin(ka\cos\beta)\sin(kb\sin\beta)/\sin\beta\cos\beta. \tag{2.26}$$

This simple expression enables an estimate to be made of the influence of shape and orientation of a single rectangular pressure distribution on the maximum power capture width.

Notice that from (2.25)

$$l_{\max}(\beta_1)/l_{\max}(\beta_2) = |f(\beta_1)|^2/|f(\beta_2)|^2,$$

so that in particular

$$l_{\max}(\tfrac{1}{2}\pi)/l_{\max}(0) = a^2\sin^2 kb/b^2\sin^2 ka \tag{2.27}$$

revealing the relative effectiveness of a pressure surface in beam and head seas.

Results based on the computation of equations (2.25)–(2.27) are given in figures

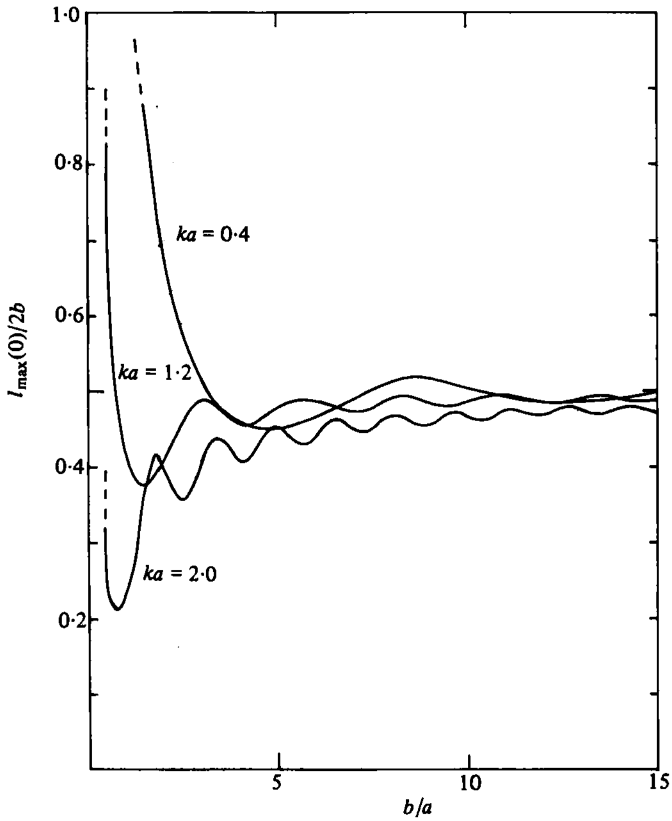

FIGURE 2. Variation of maximum capture width ratio, $l_{max}/2b$, with aspect ratio, b/a, for regular waves approaching a rectangular absorbing surface pressure distribution, for different values of dimensionless wavenumber ka.

1 and 2. In figure 1 the capture width, non-dimensionalized with respect to the width of the device, $2b$, is sketched as a function of incident wave angle, for the case of $b/a = 2$, and for different values of ka. Not shown is the case $b = a$, where the variation of $l_{max}/2b$ from the axisymmetric result (2.24) is remarkably small with the optimum angle of incidence being $\beta = \frac{1}{4}\pi$ when the wave crests are parallel to a diagonal of the square. As might be expected the fluctuations of $l_{max}/2b$ with β are larger for larger ka since the rectangular shape has more influence on the shorter waves. For instance an axisymmetric pressure distribution has a maximum capture width of about $\frac{5}{8}$ of a diameter in waves of about 8 times the diameter ($ka = 0{\cdot}4$). For a rectangular distribution of the same width but half the length ($b/a = 2$) the increase in capture width in beam seas, $\beta = 0°$, is only about 10 %. On the other hand for waves of 4 times the diameter ($ka = 0{\cdot}8$) the capture width increases by about 60 % from $\frac{3}{10}$ of a diameter to over $\frac{2}{5}$ of the width of the device. Because of the form of (2.26) we find that the opposite effect occurs in head seas, $\beta = \frac{1}{2}\pi$, where the rectangular distribution (with $b/a \rightarrow 1$) is always less efficient than the axisymmetric distribution. Just how the capture width ratio in beam seas depends upon the aspect ratio (b/a) of the rectangular distribution is shown in figure 2 for different values of ka. As might be expected, as $b/a \rightarrow \infty$ the capture width ratio approaches 0·5, being

the result for the efficiency of power absorption by a two-dimensional distribution (see equation (2.29)).

Results appropriate to *two-dimensional* pressure distributions can be obtained by returning to (2.21) and using the results (A 32), (A 33) which give

$$B_{mn} = \frac{1}{8P_W}\{q_{dm}(\pi)\, q^*_{dn}(\pi) + q_{dm}(0)\, q^*_{dn}(0)\}. \tag{2.28}$$

Again, as for three-dimensional pressure distributions, in order to estimate the maximum efficiency, it is only necessary to solve a single diffraction problem for the scattering of a wave-train by the rigid part of the system of pressure surfaces. Once this is determined, q_{dm} follows from (A 17) and the maximum power absorbed from (2.21).

Considerable simplification follows from considering the case of a single pressure surface, and we obtain an *efficiency* of power absorption

$$\eta_{\max} = W_{\max}/P_W = |q_d(0)|^2/(|q_d(0)|^2 + |q_d(\pi)|^2). \tag{2.29}$$

Here $W_{\max}$ is to be interpreted as the maximum mean power absorbed per unit width of the pressure distribution.

For waves approaching from $x = +\infty$ the argument of the numerator must be replaced by π. An alternative expression is

$$\eta_{\max}(0) = 1/(1 + |f^+/f^-|^2) \tag{2.30}$$

(where $f^\pm$ are defined by (A 28)), showing, just as for the rigid wave-energy absorber in two dimensions, that a good unidirectional wave generator (in the direction from where the incident wave comes) is a good absorber.

Again from (2.30) it follows that for a pressure distribution which is symmetric about the x-axis, so that $f^+ = f^-$, the maximum efficiency is $\frac{1}{2}$, whilst from (2.30) for an arbitrary single pressure distribution

$$\eta_{\max}(0) + \eta_{\max}(\pi) = 1.$$

For the next simplest case of two pressure distributions ($N = 2$), substitution of (2.28) into (2.21) gives, after some algebra,

$$\eta_{\max}(0) = \eta_{\max}(\pi) = 1,$$

showing that all the incident wave-energy can be absorbed. This follows provided

$$q_{d1}(\pi)\, q_{d2}(0) \neq q_{d1}(\pi)\, q_{d2}(0), \tag{2.31}$$

which is the condition which ensures that $\mathbf{B}^{-1}$ exists. From (A 34) equation (2.31) is seen to be equivalent to

$$f_1^+ f_2^- \neq f_2^+ f_1^-,$$

a condition occurring in the rigid-body case in both Srokosz & Evans (1979) and Count & Jefferys (1980) who point out that this excludes both modes being either symmetric ($f_m^+ = f_m^-$) or anti-symmetric ($f_m^+ = -f_m^-$).

If $N > 2$ the formula (2.14) no longer applies since $\mathbf{B}$ is automatically singular. This follows since $\mathbf{B}$ can be written

$$\mathbf{B} = \frac{1}{8P_W}\{\mathbf{q}_d(\pi)\, \mathbf{q}^*_d(\pi) + \mathbf{q}_d(0)\, \mathbf{q}^*_d(0)\}, \tag{2.32}$$

showing that $\mathbf{B}$ can be of rank 2 at most. This point is also made by Count & Jefferys (1980) in the rigid-body context.

The form (2.32) provides an alternative expression to (2.12) for the mean power, namely

$$W = \tfrac{1}{2}\mathscr{R}\mathbf{p}^*\mathbf{q}_d - \frac{1}{16P_W}\{|\mathbf{p}^*\mathbf{q}_d(\pi)|^2 + |\mathbf{p}^*\mathbf{q}_d(0)|^2\} \tag{2.33}$$

which is analogous to that derived by Newman (1976), equation (61a) for the case of a rigid body oscillating in a number of modes.

It seems likely that the maximum absorption efficiency for $N > 2$ is also unity but a general proof from (2.33) is not yet available. It is clear, however, that the optimal values for $\mathbf{p}$ will not be unique.

3. Conditions for resonance

The maximum absorbed power has been shown to be given by (2.21). However, this can only be achieved if we can arrange that $\mathbf{\Lambda} = \bar{\mathbf{Z}}$. In practice, it is unlikely that the matrix $\mathbf{\Lambda}$ will be other than real and diagonal with positive elements. We consider the implications of this for the case of a single internal free surface.

We have

$$W = \frac{1}{8}\frac{|q_d|^2}{B}\left\{1 - \frac{|\lambda - \bar{Z}|^2}{|\lambda + Z|^2}\right\} \tag{3.1}$$

$$= \frac{1}{8}\frac{|q_d|^2}{B}\left\{1 - \frac{(\lambda - B)^2 + \omega^2 A^2}{(\lambda + B)^2 + \omega^2 A^2}\right\}. \tag{3.2}$$

For given A, B, as functions of $\omega^2 a/g$,

$$\lambda_{\text{opt}} = (B^2 + \omega^2 A^2)^{\frac{1}{2}} \tag{3.3}$$

with

$$W_{\max} = \frac{1}{8}\frac{|q_d|^2}{B}\left\{1 - \left(\frac{\lambda_{\text{opt}} - B}{\lambda_{\text{opt}} + B}\right)\right\}. \tag{3.4}$$

For an axisymmetric pressure distribution and associated structure, from (A 27)

$$B = 2\pi |q_d|^2 / 8\lambda P_W$$

so that

$$kl_{\max} = 1 - \left(\frac{\lambda_{\text{opt}} - B}{\lambda_{\text{opt}} + B}\right)$$

$$= 2\{1 + (1 + \omega^2 A^2/B^2)^{\frac{1}{2}}\}^{-1} \tag{3.5}$$

whilst, for a two-dimensional symmetric pressure distribution,

$$\eta_{\max} = \{1 + (1 + \omega^2 A^2/B^2)^{\frac{1}{2}}\}^{-1}. \tag{3.6}$$

It is clearly of interest to ascertain whether there are values of $\omega^2 a/g$ for which A vanishes, corresponding to the induced volume flux downwards across the surface being exactly in phase with the applied pressure. Two simple examples will be considered for which explicit solutions can be obtained.

(a) The two-dimensional wave field created by a uniform simple harmonic pressure over the finite interval $|x| < a$ of the x-axis representing the free surface was first

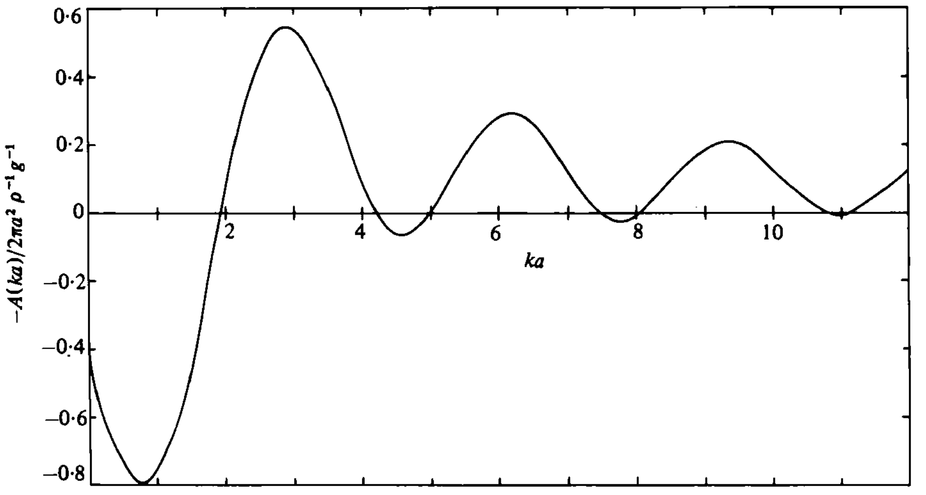

FIGURE 3. Variation of the function $A(ka)$, defined by equation (3.10), with dimensionless wavenumber ka, for a circular uniform oscillatory surface pressure distribution of radius a.

solved by Stoker (1957) and subsequently considered by Ogilvie (1969) and Falcão & Sarmento (1980). For this simple problem we have, from (A 32), (A 33)

$$B(ka) = |q_d|^2/4P_W, \tag{3.7}$$

and it is easily shown that

$$q_d = 2gA\omega^{-1}\sin ka, \tag{3.8}$$

since there is no scattered potential.

The expression for $A(ka)$ is more complicated, involving the special functions Ci and Si. However Falcão & Sarmento (1980) have shown, and it can be confirmed from Ogilvie (1969), figure 15, that $A(ka) = 0$ for $ka \doteqdot 1{\cdot}3$ corresponding to a strip half-width of about one-fifth of a wavelength. Equation (3.6) now shows that, as ka increases from zero, the maximum efficiency of 0·5 is achieved at about $ka = 1{\cdot}3$, when $A(ka) = 0$, but the efficiency drops to zero at subsequent values of ka for which $B(ka) = 0$, namely $ka = n\pi$, $n = 1, 2, \ldots$. Curves showing the variation of $\eta_{\max}$ with ka are given by Falcão & Sarmento (1980).

(*b*) As a further example we consider the axisymmetric extension of the above to a uniform oscillatory pressure distribution over a disk of radius a on the free surface in deep water. The resulting three-dimensional axisymmetric wave field can be determined explicitly either by using Green's theorem in conjunction with the fundamental wave source potential in three dimensions or, more simply, by use of Hankel transforms.

It is found that

$$B(ka) = 2\pi^2a^2\omega\rho^{-1}g^{-1}J_1^2(ka), \tag{3.9}$$

whilst

$$A(ka) = -2\pi a^2\rho^{-1}g^{-1}\left\{\pi J_1(ka)\,Y_1(ka) + 2\pi^{-1}ka\int_0^\infty \frac{I_1(u)\,K_1(u)}{u^2+k^2a^2}\,du\right\}. \tag{3.10}$$

Here J_1, Y_1, I_1, K_1 are Bessel functions in the usual notation. A derivation of this result together with extensions to finite depth and surface-piercing ducts can be found in Thomas (1981).

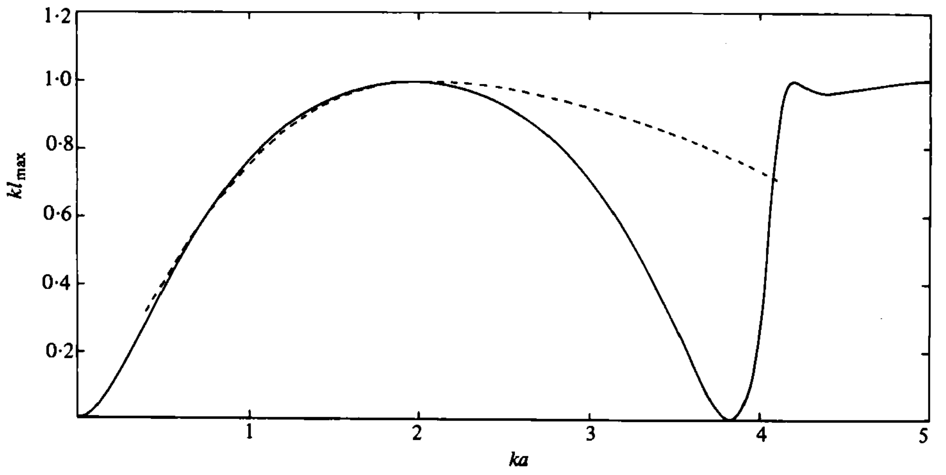

FIGURE 4. Variation of kl_{max} with dimensionless wavenumber ka, for a circular absorbing oscillatory surface pressure distribution (solid line) and a circular absorbing oscillatory rigid disk (dotted line).

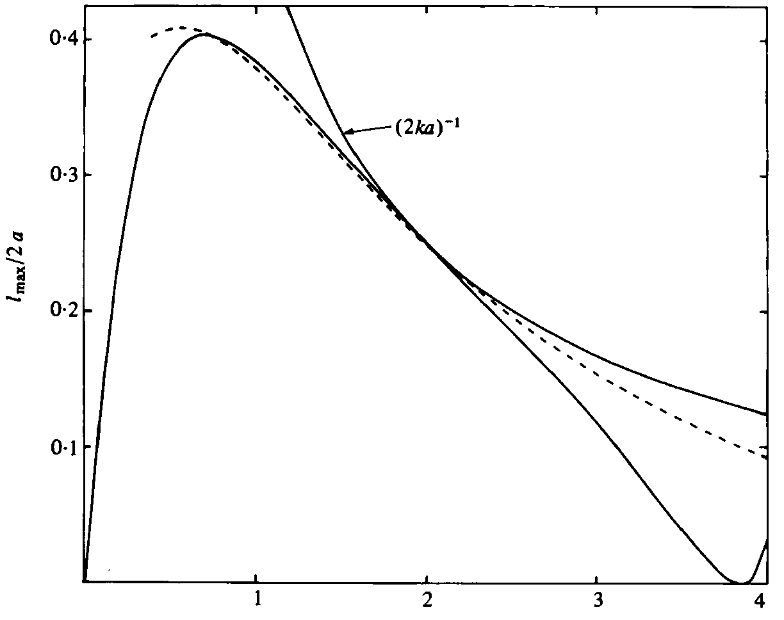

FIGURE 5. Variation of dimensionless capture width ratio $l_{max}/2a$ with dimensionless wavenumber for a circular absorbing oscillatory surface pressure distribution (solid line) and a circular absorbing oscillatory rigid disk (dotted line). Also shown is the theoretical optimum $(2ka)^{-1}$ in each case.

A graph of the expression in curly brackets in (3.10) against ka is shown in figure 3. It appears that $A(ka)$ has just seven zeros, the first of which is $ka = 1{\cdot}96$ corresponding to a disk radius of about three tenths of a wavelength. The first zero of $B(ka)$ occurs at $ka = 3{\cdot}83$.

Figure 4 shows the variation of kl_{max} with ka whilst figure 5 shows the variation of the capture width non-dimensionalized with respect to the disk diameter. It can be seen that the maximum value of $kl_{\max}$ occurs at the first zero of $A(ka)$ whilst $kl_{\max}$ is reduced to zero at $ka = 3{\cdot}83$ corresponding to the first zero of $B(ka)$. Subsequent zeros of $A(ka)$, $B(ka)$ give rise to oscillatory behaviour of $kl_{\max}$ as ka increases. It is perhaps more illuminating to look at the capture width ratio $l_{\max}/2a$ as it varies with ka. The effect of the term involving $A(ka)$ is to give an absolute maximum to the capture width ratio of about 0·4 in the range of interest at $ka \doteqdot 0{\cdot}7$ or a wavelength to diameter ratio of about 5. Also shown is the curve $(2ka)^{-1}$ obtained from (2.24) by assuming that resonance can be achieved at all frequencies. As expected the only point of contact with the $l_{\max}/2a$ curve is at the first zero of $A(ka)$.

4. Comparison with a rigid plate model

Previous treatments of wave-energy devices which are based on the idea of forcing a trapped air volume at the water surface through a turbine, have modelled the free-surface by a rigid surface condition of prescribed vertical velocity. See, for example, Count *et al.* (1981). It is argued that for long waves at least ($k \rightarrow 0$), the pressure condition (2.6) reduces to the rigid surface condition. However, to be consistent, the same approximation should be made over the entire free surface, in which case the wave element of the problem is lost.

It is of interest to look at the differences in the results of the two approaches. We consider only a single symmetric device consisting of either a rigid surface plate or a pressure distribution. In both cases the maximum efficiency is $\frac{1}{2}$ in two dimensions, or the maximum capture width is k^{-1} for axisymmetric devices.

The actual capture width for a circular absorbing plate can be written (Evans 1976) in a form precisely analogous to (3.4), (3.5) where now λ is the positive velocity-proportional damping coefficient connecting the vertical externally applied opposing force on the disk and the vertical velocity of the disk. Furthermore

$$A(\omega) = m + a_{33}(\omega) - c\omega^{-2}, \tag{4.1}$$

where m is the mass of the disk, $a_{33}(\omega)$ its frequency-dependent heave added mass, and $c = \pi a^2 \rho g$ is the buoyancy-restoring coefficient under the assumption that the disk on the surface is part of a finite-length circular cylinder extending above the surface. The coefficient $B(\omega) = b_{33}(\omega)$, the frequency-dependent damping coefficient for the *forced* oscillatory heave motion of the disk with unit velocity amplitude.

Now since the disk is assumed to lie on the free surface its mass can be ignored compared to its added mass, which in turn can be non-dimensionalized by writing $a_{33}(\omega) = 2\pi\rho a^3 \mu_3(\omega)$ so that

$$A(\omega) = 2\pi\rho a^3 \{\mu_3(\omega) - (2ka)^{-1}\}. \tag{4.2}$$

Similarly $B(\omega) = 2\pi\rho a^3 \omega \lambda_3(\omega)$ where $\lambda_3(\omega)$ is the non-dimensional damping coefficient.

The heave added mass and damping coefficients for a circular dock in the surface have been determined by McCamy (1961), although there appears to be a typographical error in the labelling of the ordinates in his figures 6 and 7; the asymptotic result $\mu_3 \sim 2/3\pi$ as $ka \rightarrow \infty$ together with other information suggests that the values given for μ_3 and λ_3 need to be reduced by a factor 2π. With this correction, it is found that $A(\omega)$ vanishes once only at $ka \doteqdot 2{\cdot}1$ whilst $B(\omega)$ is always positive. The effect of these differences on $kl_{\max}$ and $l_{\max}/2a$ are shown by the dotted lines in figures 4 and 5.

It can be seen from figure 4 that, in the range $0 < ka < 4$ which encompasses the range of practical interest for wave-energy devices, the major difference in kl_{max} occurs for $ka > 2$ where the pressure distribution values begin to fall, reaching zero at the value of $ka \doteqdot 3{\cdot}82$ corresponding to the first zero of $B(ka)$ for this case. Since $B(ka)$ is never zero for the rigid surface dock, no such fall in kl_{max} occurs in this case. The same is true for the capture width ratios in figure 5; in fact over the range of wavelength/diameter ratios from 1·5 to 4 the differences in the two capture width ratios are small.

Similar differences occur in the case of the two-dimensional strip also considered by McCamy (1961). In this case $c = 2a\rho g$ in (4.1) and $A(\omega)$ vanishes for $ka(=\omega^2 a/g) \doteqdot 1{\cdot}42$ compared with the value 1·3 predicted by Falcão & Sarmento (1980).

5. Conclusion

A number of problems relating to the absorption of wave energy by oscillatory uniform surface pressure distributions have been considered. It has been shown, using linearized water-wave theory, that general expressions can be derived for the mean power absorbed by an arbitrary system of pressure distributions in terms of: an admittance matrix relating volume flux to applied pressure for the system, the induced volume flux due to the incident and scattered potential alone, and the (assumed linear) pressure–volume flux characteristics of the power-take-off mechanism.

It has been further shown that under perfect impedance matching the maximum mean power absorbed can be determined solely by solving a linear wave-diffraction problem common in ship hydrodynamic theory without reference to pressure distributions. This follows from new results relating the required damping coefficients for given pressure distributions to the induced volume flux arising from the diffraction problem. Most of these results are derived in the appendix.

In the more likely case of imperfect matching, it has been shown that, for single pressure distributions in either two or three dimensions, conditions for resonance exist which attach a size of the pressure distribution to the incident wavelength. A comparison with the resonant conditions for comparable rigid body wave-energy devices shows that only slight differences occur for values of ka within the range of practical interest, suggesting that the use of such rigid body models for devices which depend upon the surface pressure idea for their operation will provide satisfactory results. For larger ka, however, significant differences do occur and this will be of importance in nonlinear problems where high-frequency Fourier components are considered. In general, however, there can be little justification in future for using rigid-body theory rather than the present theory for such devices, since in addition to more accurately describing the physical situation, it also has the advantage of producing simpler boundary-value problems to be solved.

The present paper only attempts to open up the theory of energy-absorbing pressure distributions and there are clearly a number of problems which need to be tackled. The greatest drawback to the present theory is the assumption of a *linear* turbine characteristic at each pressure distribution. In fact it is more likely to be quadratic in form (Fry & Jefferys 1979). Again the assumption of incompressibility of the enclosed air volume needs to be examined. Both these points have been described by Falcão & Sarmento (1980) and it appears likely that, using appropriate Fourier series expansions, they could be incorporated into the present general theory.

A further direction of study would be to apply the general theory to a typical Kaimei device to obtain estimates of optimum capture widths and chamber pressures. It would be more sensible, perhaps, to delay such an application until the theory has been extended to include genuine nonlinear turbine characteristics.

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Appendix. Reciprocal relations involving pressure distributions

The form of (2.14) suggests, by analogy with the theory for arrays of rigid energy-absorbing bodies, that the relation

$$B_{mn} = \frac{1}{8\lambda P_W}\int_0^{2\pi} q_{dm}(\theta)\, q_{dn}^*(\theta)\, d\theta \tag{A 1}$$

holds between the elements of the damping matrix $\mathbf{B}$ and the elements of the diffracted wave induced volume flux vector $\mathbf{q}_d$. In addition to (A 1) further relations are derived in this appendix appropriate to free-surface pressure distributions. The method of derivation follows closely that used by Newman (1976) to obtain the corresponding rigid-body results. They are presented in some detail here as they appear to be new and they may have applications in other contexts such as the study of air-cushion vehicles.

Referring to the main body of the paper, the potential $\psi(x, y, z)$ satisfies

$$\nabla^2\psi = 0 \quad \text{in the fluid;} \tag{A 2}$$

$$\partial\psi/\partial n = 0 \quad \text{on } S_B, \text{ the fixed rigid boundary;} \tag{A 3}$$

$$\frac{\partial\psi}{\partial z} - k\psi = \begin{cases} i\omega\rho^{-1}g^{-1}p_i & \text{on } S_i, \text{ the } i\text{th internal free surface,} \quad \text{(A 4)} \\ 0 & \text{on } S_F, \text{ the external free surface.} \quad \text{(A 5)} \end{cases}$$

At large distances ψ behaves like an *out*going wave potential. We have (A 6)

$$q_i = \int_{S_i} \frac{\partial\psi}{\partial z}\, dS, \tag{A 7}$$

the time-independent, complex, volume flux across S_i, and, from (2.11),

$$q_i = -\sum_{j=1}^{N} Z_{ij} p_j, \tag{A 8}$$

where

$$\mathbf{Z} = \{Z_{ij}\} = \mathbf{B} - i\omega\mathbf{A}. \tag{A 9}$$

Let

$$\psi = \sum_{j=1}^{N} i\omega\rho^{-1}g^{-1}p_j\psi_j, \tag{A 10}$$

where the ψ_j satisfy (A 2), (A 3), (A 6) and

$$\frac{\partial\psi_j}{\partial z} - k\psi_j = \begin{cases}\delta_{ij} & \text{on} \quad S_i, \\ 0 & \text{on} \quad S_F.\end{cases} \tag{A 11}$$

It follows from (A 7), (A 8), (A 10) that

$$Z_{ij} = -i\omega\rho^{-1}g^{-1}\int_{S_i}(\partial\psi_j/\partial z)\,dS \tag{A 12}$$

so that, in particular,

$$\mathscr{I}\int_{S_i}(\partial\psi_j/\partial z)\,dS = \rho g\omega^{-1}B_{ij}, \tag{A 13}$$

$$\mathscr{R}\int_{S_i}(\partial\psi_j/\partial z)\,dS = \rho g A_{ij}. \tag{A 14}$$

Finally, we assume that

$$\psi_i \sim f_i(\theta)\,e^{ikR}\,e^{kz}/(kR)^{\frac{1}{2}} \quad \text{as} \quad R\rightarrow\infty, \quad R = (x^2+y^2)^{\frac{1}{2}}. \tag{A 15}$$

The diffraction potential is

$$\phi_d = \phi_0 + \phi_s,$$

where

$$\phi_0 = gA\omega^{-1}\exp\{ik(x\cos\beta + y\sin\beta) + kz\} \tag{A 16}$$

satisfying $\partial\phi_d/\partial z = 0$ on S_B, the fixed rigid surfaces, and

$$q_{di} = \int_{S_i}(\partial\phi_d/\partial z)\,dS. \tag{A 17}$$

Also

$$\phi_s \sim f_s(\theta)\exp\{ikR + kz\}/(kR)^{\frac{1}{2}} \quad \text{as} \quad R\rightarrow\infty \tag{A 18}$$

and both ϕ_0 and ϕ_s satisfy (A 5).

Now if ϕ, ψ are any two sufficiently regular harmonic functions in a given region,

$$\int_S\left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\psi}{\partial n}\right)dS, \tag{A 19}$$

where the surface integral is taken over any *closed* surface containing the region.

In particular, consider the surface integral

$$I(\psi_i,\psi_j) = \int_{S_i \cup S_j \cup S_B}\left(\psi_i\frac{\partial\psi_j}{\partial n} - \psi_j\frac{\partial\psi_i}{\partial n}\right)dS, \tag{A 20}$$

where the integration is over both the ith and jth internal free surface, and also over the rigid boundary S_B. Now the surface of integration in (A 20) can be closed by a large, vertical circular cylinder S_C enclosing *all* free surfaces and rigid boundaries and extending from the free surface downwards. Since both ψ_i, ψ_j behave like outgoing waves at large distances no extra contribution to (A 20) is obtained from the integral over S_C. Also, there is no contribution from integrating over S_F or over S_k, $k = 1, \ldots, N$, $k \neq i$, $k \neq j$. Hence it follows from (A 19) that

$$I(\psi_i,\psi_j) = 0 \quad \text{for all} \quad i,j. \tag{A 21}$$

Again the contribution to (A 20) from S_B vanishes by virtue of condition (A 3) and so from (A 20), (A 11)

$$0 = \int_{S_i} + \int_{S_j} = \int_{S_i} -\psi_j \, dS + \int_{S_j} \psi_i \, dS,$$

showing that

$$Z_{ij} = Z_{ji} \quad \text{for all} \quad i,j. \tag{A 22}$$

It is convenient at this stage, following Newman (1976), to introduce the Kochin function

$$H_i(\theta) = -k \int_{S_B \cup S_i} \left(\frac{\partial \psi_i}{\partial n} - \psi_i \frac{\partial}{\partial n} \right) \exp\{-ik(x\cos\theta + y\sin\theta) + kz\} \, dS. \tag{A 23}$$

Since both ψ_i and $\exp\{-ik(x\cos\theta+y\sin\theta)+kz\}$ are harmonic functions satisfying (A 5), it follows from (A 19) that $H_i(\theta)$ may be written as the *negative* of the same integrand integrated over S_C. This enables the far-field behaviour of ψ_i given by (A 15) to be used to obtain

$$H_i(\theta) = \tfrac{1}{2} i (kR)^{\frac{1}{2}} \int_0^{2\pi} f_i(\theta') \{1 + \cos(\theta - \theta')\} \exp\{ikR(1 - \cos(\theta - \theta'))\} \, d\theta',$$

where $R = (x^2+y^2)^{\frac{1}{2}}$. If, now, $R \to \infty$, it follows from the method of stationary phase that

$$H_i(\theta) = i(2\pi)^{\frac{1}{2}} f_i(\theta) \exp(\tfrac{1}{4} i\pi) \tag{A 24}$$

so that the Kochin function is directly related to the far-field radiated amplitude of the potential ψ_i.

Now, by direct substitution,

$$H_i(\beta + \pi) = -k\omega g^{-1} A^{-1} \int_{S_B \cup S_i} \left(\frac{\partial \psi_i}{\partial n} - \psi_i \frac{\partial}{\partial n} \right) \phi_0(x,y,z) \, dS$$

with ϕ_0 given by (A 16).

We can replace ϕ_0 by $\phi_d - \phi_s$. But the surface integral involving ϕ_s vanishes since if, using (A 19), we replace it by the negative of a surface integral over S_j and S_C, we find that there is no contribution from either since both ψ_i and ϕ_s satisfy conditions (A 5) and (A 6).

We are left with

$$H_i(\beta + \pi) = -k\omega g^{-1} A^{-1} \int_{S_i} \left(\frac{\partial \psi_i}{\partial n} - \psi_i \frac{\partial}{\partial n} \right) \phi_d(x,y,z) \, dS$$

since both ψ_i and ϕ_d satisfy (A 3).

From (A 11), then,

$$\begin{aligned} H_i(\beta + \pi) &= -k\omega g^{-1} A^{-1} \int_{S_i} \{(k\psi_i + 1)\phi_d - \psi_i k\phi_d\} \, dS \\ &= -k\omega g^{-1} A^{-1} \int_{S_i} \phi_d \, dS = -\omega g^{-1} A^{-1} \int_{S_i} (\partial\phi_d/\partial z) \, dS \\ &= -\omega g^{-1} A^{-1} q_{di}(\beta). \end{aligned} \tag{A 25}$$

Bearing in mind (A 24), we see that the volume flux across S_i due to the incident plus diffracted wave fields is proportional to the far-field behaviour of the radiation potential in the *opposite* direction to that of the incident wave-train, owing to the uniform oscillatory pressure distribution across S_i. This result corresponds to the Haskind relations for rigid-body motions (Newman 1976, equation (45)).

Next, we consider

$$I(\psi_i, \psi_j^*) = \int_{S_i \cup S_j \cup S_B} (\psi_i\, \partial\psi_j^*/\partial n - \psi_j^*\, \partial\psi_i/\partial n)\, dS,$$

where $*$ denotes the complex conjugate. Since both ψ_i, ψ^*_j satisfy (A 3) we have

$$\begin{aligned} I(\psi_i, \psi_j^*) &= \int_{S_i \cup S_j} \dots = \int_{S_i} -\psi_j^*\, dS + \int_{S_j} \psi_i\, dS \\ &= 2i \mathscr{I} \int_{S_i} \psi_j\, dS = 2ik^{-1} \mathscr{I} \int_{S_i} \frac{\partial \psi_j}{\partial z}\, dS \\ &= 2ik^{-1} \rho g \omega^{-1} B_{ij}, \end{aligned}$$

where (A 11), (A 13), (A 22) have been used.

Again

$$\begin{aligned} I(\psi_i, \psi^*) &= -\int_{S_0} (\psi_i\, \partial\psi_j^*/\partial n - \psi_j^*\, \partial\psi_i/\partial n)\, dS \\ &= \frac{i}{2\pi k} \int_0^{2\pi} H_i(\theta)\, H_j^*(\theta)\, d\theta, \end{aligned} \tag{A 26}$$

where (A 15) and (A 24) have been used.

Hence

$$\begin{aligned} B_{ij} &= \frac{\omega}{4\pi\rho g} \int_0^{2\pi} H_i(\theta)\, H_j^*(\theta)\, d\theta \\ &= \frac{1}{8\lambda P_W} \int_0^{2\pi} q_{di}(\theta)\, q_{dj}^*(\theta)\, d\theta \end{aligned} \tag{A 27}$$

in which (A 25) has been used and a simple change of variable of integration made. Here $P_W = \frac{1}{4}\omega^{-1}\rho g^2 A^2$.

In addition to these three-dimensional results it is also possible to derive important two-dimensional reciprocal relations for pressure distributions. Equations (A 2) to (A 14) remain the same, but now (A 15) must be replaced by

$$\psi_i \sim f_i^{(\pm)} \exp(kz \pm ikx) \quad \text{as} \quad x \to \pm\infty \tag{A 28}$$

where the motion takes place in the (x, z) plane. Similarly (A 16) becomes

$$\phi_0 = gA\omega^{-1} \exp(ikx + kz), \tag{A 29}$$

and

$$\phi_s \sim f_s^{(\pm)} \exp(kz \pm ikx) \quad \text{as} \quad x \to \pm\infty \tag{A 30}$$

and the Kochin function is now only defined for angles β equal to 0 or π. The preceding arguments go through with little change and the results in two dimensions corresponding to (A 24), (A 25), (A 26) are

$$H_i \begin{pmatrix} 0 \\ \pi \end{pmatrix} = ikf_i^{(\pm)}, \tag{A 31}$$

$$J_i \begin{pmatrix} 0 \\ \pi \end{pmatrix} = -\omega g^{-1} A^{-1} q_{di} \begin{pmatrix} \pi \\ 0 \end{pmatrix}, \tag{A 32}$$

$$B_{ij} = \frac{1}{2\rho\omega} \{H_i(0)\, H_j^*(0) + H_i(\pi)\, H_j^*(\pi)\}. \tag{A 33}$$

The equation for $H_i(0)$ in (A 32) corresponds to an incident wave $gA\omega^{-1}\exp[-ikx+kz]$ from $x=+\infty$, with $q_{di}(\pi)$ being the corresponding volume flux across S_i.

It follows from (A 31) and (A 32) that

$$q_{di}\begin{pmatrix}\pi\\0\end{pmatrix} = -i\omega A f_i^{(\pm)} \tag{A 34}$$

showing that in two dimensions also the induced volume flux across S_i due to the incident and diffracted field is proportional to the far-field amplitude of the radiated potential in the direction from which the incident wave comes. In (A 34) q_{di} is the volume flux per unit breadth of the surface pressure. Further relations between the properties of the solution ψ_i to the forced radiation problem and ϕ_d, the solution to the diffraction problem in both two and three dimensions can also be derived via the use of Kochin functions and Green's theorem. In particular the new relations proved by Newman (1976), equations (48), (49)), carry over to pressure distributions without change. Since the method of proof is identical and since they are not needed in the present context they are not given here.

It follows that all of the results relating properties of the *forced* motion of a rigid body in a given mode (Newman 1976), or of a number of independently oscillating rigid bodies (Srokosz 1979), to the corresponding diffraction problem of the scattering of an incident wave field by such a body or bodies, have their counterpart in surface pressure distributions. The correspondence follows if the rigid bodies are regarded as thin horizontal plates making unit vertical oscillations in the free surface. Although k is required to be zero in (A 11) in this case, this does not affect results derived using Green's theorem. The correspondence is completed by noting that the vertical exciting force on S_i, is just $i\omega\rho k^{-1}q_{di}$, that is, proportional to the volume flux across S_i.

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